Propagation Path Loss Prediction Using Parabolic Equations for Narrow and Wide Angles

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I. INTRODUCTION

For the planning of any mobile communications system it is important to know the behavior of electromagnetic waves propagating in the studied region. This propagating is affected by a number of factors that may be of natural origin such as lightning, rain, vegetation or provoked by man as the effects of buildings, the noise of engines, transmission lines and other. For a mixed-path environment, the electromagnetic waves suffer a great influence of the effect of multiscattering, where this effect causes significant attenuation in the mobile radio signal level, which are due the variations of the electrical properties of vegetation (leaves, changes) and soil. Thus, for a perfect study of the coverage area of mobile communications system, there is need to develop a model to predict the losses occurred in the of signal propagation within the studied environments.

Parabolic equation solvers provide a powerful modeling capability for propagation of electromagnetic waves over long distances in complex environments. They are used extensively for predicting radar coverage in ducting environments over rough surfaces. Two primary classes of solvers are used in conjunction with the parabolic wave equation. The first is a finite-difference approach that relies on fine discretization of the spatial domain for accurate representation of the field propagation. The second is the split-step approach that implements propagation in the Fourier domain and utilizes a series of “phase-screens” to account for refraction effects [1, 2].

It was natural therefore for Egli to produce a model based on plane-earth propagation, after observer that there was a tendency for the median signal strength in a small area to follow an inverse fourth-power law with range from the transmitter. However he also observed firstly that there was an excess loss over and above predicted and secondly that this excess loss depended upon frequency and the nature of the terrain [3].

The Lee model is a power law model, which takes into account the antenna height of the base station and the variation in terrain where the effective base station antenna height is determined by the projection of slope terrain in near vicinity of the mobile to the base station location [4].

This paper presents a model for calculating the propagation loss of electromagnetic waves based on the formalism of parabolic equations [1], which has the great advantage of reduced computational effort to calculate and small margins of error.

To validate the proposed model, this paper presents the results of measurement campaigns conducted in three cities in Pará State (Brazil), where were used the frequencies of 900 MHz and 1.8 GHz for the tests, values used in mobile radio in Brazil. These values were compared with measurements obtained using the prediction model of parabolic equations and with Egli Model and Lee Model, models existing in the literature.

Initially, the environment was modeled considering streets with buildings and vegetation, and then, we applied the method of parabolic equations for the calculation of electric fields considering the electrical parameters involved in the simulated environment. To solve the resulting parabolic equation, the finite difference scheme of Crank-Nicolson method was used for narrow angles, up to 15°. For wide angles, up to 90°, the mixed Fourier transform was used.

This paper is organized as it follows: the propagation model, the method of parabolic equation (PE) and the mixed Fourier transform, is described in section II; section III describes the environments; in section IV are presented the path loss models; on section V shows the results; and section VI, the conclusion.

II. THE THEORETICAL METHOD

The two-dimensional scalar wave equation can written as [5]
where $k$ is the wave number and $n$ the refractive index. Following Levy [6], we choose $x$ as the paraxial direction and replace the function $\psi(x,z)$ by $e^{ik}E(x,z)$ yielding the scalar equation [5]

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial z^2} + 2ik \frac{\partial \psi}{\partial x} + k^2 \left(n^2 - 1\right) \psi = 0$$

(1)

This equation can be formally written as

$$\left(\frac{\partial}{\partial x} + ik \left(1 - Q\right)\right) \left(\frac{\partial}{\partial x} + ik \left(1 + Q\right)\right) E = 0$$

(2)

where $Q = \sqrt{1 + \psi^2/\psi^2_n + n^2(x,z)} = \sqrt{1 + Z}$ [5]. One may here note that the operator $Z$ would represent a quantity that is small compared to one.

Equation (3) represents both forward and backward propagating waves and the part that represents forward propagating waves is

$$\left(\frac{\partial}{\partial x} + ik \left(1 - Q\right)\right) E = \left[\frac{\partial}{\partial x} + ik \left(1 - \sqrt{1 + Z}\right)\right] E = 0$$

(4)

The simplest approximation of (4) is obtained by using first-order Taylor expansions of the square-root and exponential functions. This yields the standard parabolic equation (SPE) [6]

$$\frac{\partial^2 E}{\partial z^2} + 2ik \frac{\partial E}{\partial x} + k^2 \left(n^2 - 1\right) E = 0$$

(5)

This is the parabolic equation used for narrow angle (NA) in this paper. The error in (5) is going from $10^{-3}$ for an angle of 1°, to $10^{-3}$ for an angle of 10° and over $10^{-2}$ for an angle of 20° [6].

In this paper was used the finite difference scheme of Crank-Nicolson applied to the standard parabolic equation. The approach of the central finite differences was calculated for the derivatives of first and second order in $x$ and $z$, where $\xi_m = (x_{m-1} + x_m)/2$ is the midpoint in the solution from $x_{m-1}$ to $x_m$ range. Using $E_m^n = E(x_m, z_j)$, $b = 4ik \left(\Delta z^2 / \Delta x\right)$ and $a_m^n = k^2 \left(n^2 \left(\xi_m, z_j\right) - 1\right) \Delta z^2$ , and applying in (5) we obtain [6]

$$E^n_j \left(-2 + b + a_m^n\right) + E^{n+1}_j + E^{n-1}_j = E^{n+1}_j \left(2 + b - a_m^n\right) - E^{n-1}_j - E^{n-1}_j$$

(6)

and

$$Q \sim \sqrt{1 + A + \sqrt{1 + B - 1}}$$

(7)

is the new wide-angles (WA) split operator proposed by Felt and Fleck, where $A$ and $B$ are defined as: $A = \left(1/k^2\right) \left(\psi^2/\psi^2_n\right)$ and $B = n^2(z) - 1$. This approximation is exact for uniform media and the equation is only valid for commuting operators [6].

Substitution of (7) into (4) leads to the result [6]

$$\frac{\partial E}{\partial x} - i \sqrt{k^2 + \psi^2/\psi^2_n} E - ik \left(n - 2\right) E = 0$$

(8)

Equation (8) is the wide-angle parabolic equation used in this paper.

The mixed Fourier transform, implementing impedance boundary conditions, was developed to enable propagation simulation in the lower atmosphere over finitely conducting surfaces. The algorithm for the implementation of mixed Fourier transform in a discrete domain is described in [6]. The procedure to solve (8) can be written as [7]

$$E(x + \Delta x, z) = e^{ik(n-1)\Delta x/2}$$

(9)

where

$$U(x, p) = \alpha F_s \left\{e^{\alpha(n-1)\Delta x/2} E(x, z)\right\} + p F_c \left\{e^{-\alpha(n-1)\Delta x/2} E(x, z)\right\}$$

(10)

$F_s$ and $F_c$ are sine and cosine transform, respectively, $K(x)$ is defined as [7]:

$$K(x) = \left\{2\alpha \int_0^\infty f(z) e^{-\alpha z} dz; \quad \text{Re} (\alpha) > 0\right\} \quad \left\{0; \quad \text{Re} (\alpha) \leq 0\right\}$$

(11)

and the coefficient $\alpha$ represents the properties of surface medium in terms of relative complex permittivity $\eta$ [7]

$$\alpha = \frac{ik_0}{\sqrt{\eta}}$$

(12.a) vertical pol.

$$\alpha = ik_0\sqrt{\eta}$$

(12.b) horizontal pol.
III. DESCRIPTION OF THE ENVIRONMENTS

The measurements were taken in a covered by radio signal transmitted by fixed station in cities 1, 2 and 3, in Pará State. These cities are characterized by areas of dense vegetation, cut through streets paved with the presence of smaller buildings (see Fig. 1, 3, 4).

The transmitted signal was at 900 MHz, for city 1 and 2, and 1800 MHz for city 3. In the city 1, the transmitter was installed on a building in ANATEL (Agência Nacional de Telecomunicações) using a collinear antenna (see Fig. 2) with a gain of 2.14 dBi. In the city 2, the transmitter used the radio base station from OI-Cellular Company; and the antenna was an omnidirectional, with a gain of 2 dBi radiating a signal CW. In the city 3, the transmitter used the radio base station from TIM-Cellular Company with a panel antenna, with a gain of 17.5 dBi. The mobile receiver traveled at a speed of approximately 30 km/h along road inside a forest, for the three cities. The measurement results were recorded for off line processing.

For the models applications, the following set of parameters were considered and showed in the Table 1.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Symb</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>( f )</td>
<td>900 MHz</td>
</tr>
<tr>
<td>Average height forest</td>
<td>( h )</td>
<td>12 m</td>
</tr>
<tr>
<td>Transmitter height ( h_T )</td>
<td>12 m</td>
<td>70 m</td>
</tr>
<tr>
<td>Mobile receiver height ( h_R )</td>
<td>3 m</td>
<td>3 m</td>
</tr>
<tr>
<td>Receiver antenna gain ( G_R )</td>
<td>2.14 dBi</td>
<td>2.14 dBi</td>
</tr>
<tr>
<td>Transmitted power ( P_T )</td>
<td>30 dBm</td>
<td>22 dBm</td>
</tr>
<tr>
<td>Road – paved, width ( W )</td>
<td>12 m</td>
<td>11 m</td>
</tr>
<tr>
<td>Vehicle position ( w )</td>
<td>7.75 m</td>
<td>6.5 m</td>
</tr>
<tr>
<td>Forest relative permittivity [8]</td>
<td>( \varepsilon _F )</td>
<td>1.1</td>
</tr>
<tr>
<td>Forest conductivity [8]</td>
<td>( \sigma _F )</td>
<td>0.1 mS/m</td>
</tr>
<tr>
<td>Road relative permittivity [8]</td>
<td>( \varepsilon _R )</td>
<td>2.7</td>
</tr>
<tr>
<td>Road conductivity [8]</td>
<td>( \sigma _R )</td>
<td>40 mS/m</td>
</tr>
<tr>
<td>Width of the right lateral forest</td>
<td>( d_1 )</td>
<td>1500 m</td>
</tr>
<tr>
<td>Width of the left lateral forest</td>
<td>( d_2 )</td>
<td>2500 m</td>
</tr>
<tr>
<td>Transmitter distance range</td>
<td>( d )</td>
<td>500 to 5600 m</td>
</tr>
</tbody>
</table>
IV. PATH LOSS MODELS

Several theoretical and experimental models exist for to compute the path loss propagation in wireless communications, and each characterizes the environment with different view. In mixed-path, the shadowing, the scattering and the absorption caused for the vegetation can cause a significant path loss, which increases with the frequency. For the validation of the proposed model we compared the measured data and some classical models in the literature described below.

A. Egli Model

Based on a series of measurements performed over irregular terrain, Egli [9] proposes an empirical model where the attenuation of the transmitted signal depends on the law of the inverse fourth power between the transmitter and mobile receiver.

The expression for the median path loss is [10]:

\[
L_d B = 40 \log(d) + 20 \log f - 20 \log h_b - 20 \log h_m + G_b - G_m - 76.3
\]  

where \( G_b \) and \( G_m \) are the gains of transmitting and receiving antennas, \( h_b \) and \( h_m \) are the heights of the transmitter and receiver antennas, respectively, \( d \) is the distance between them and \( f \) is the frequency.

B. Lee Model

It is a point-to-point propagation model, and to get it is required three steps. The first is the creation of so-called standard conditions. To make the prediction area - point and then the prediction point to point.

The general expression of the model for the loss of the received signal is [11]:

\[
L(dB) = 123.77 + 30.5 \log d + 10n \log \frac{f}{900} - \alpha
\]  

where \( d \) is the distance between the transmitter and mobile receiver, in km, \( f \) is frequency in GHz, \( n \) is an experiment value chosen to be 3 in our simulation and \( \alpha \) is the correction factor to the standard condition in dB, given by:

\[
\alpha = \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 \; ; \quad \alpha_1 = \left( \frac{h_f}{30.48} \right)^2, \quad \alpha_2 = \left( \frac{h_b}{3} \right)^2, \quad \alpha_3 = \left( \frac{P_t}{10} \right)^2, \quad \alpha_4 = \left( \frac{G_t}{4} \right) \; \text{and} \; \alpha_5 = \frac{G_r}{G_t}, \quad \text{and choose the parameter} \; k \; \text{to be equal to} \; 2 \; [11].
\]

V. RESULTS

The data analyzed in this paper were obtained from measurement campaigns took place in the Cities 1, 2 and 3 in the State of Pará. In the Cities 1 and 2 was used the frequency of 900 MHZ and the City 3 was used the frequency of 1800 MHZ.

For the solution of the parabolic equation for narrow angles, up to 15°, it was used the finite difference method proposed by Crank and Nicolson, and for large angles, up to 90°, it was used the mixed Fourier transform. The reason for this choice was that, if the finite difference scheme was used, it would be necessary to work with penta-diagonal matrices, instead of the tri-diagonal, causing an increase in computational time [5].

To calculate the propagation path loss through the method of parabolic equations and subsequent comparison with the models of Egli and Lee was used the equation below [12]:

\[
L(dB) = 36.57 + 20 \log f + 20 \log |\mu_0| - 20 \log |\mu| - G_t - G_r
\]

where \( \mu_0 \) is the electric field at a reference distance (\( d_0 \)), \( \mu \) the electric field received, \( f \) is the frequency in GHz, and \( G_r \) and \( G_t \) are the gains of transmitting and receiving antennas in dB, respectively.

The refractive index is given by the following expression [13]:

\[
n = \left[ \frac{\varepsilon_r + \frac{i\sigma}{2\pi\varepsilon_0}}{\varepsilon_0} \right]^{\frac{1}{2}}
\]

where \( \varepsilon_r \) is the relative permittivity, \( \sigma \) is the conductivity (S/m), \( f \) is the frequency (Hz) and \( \varepsilon_0 \) is the permittivity in the vacuum (F/m).

Figures 5, 6 and 7 show the path loss in dB, depending on the distance \( d \) to the transmitter, in Kilometers, to the cities 1, 2 and 3, respectively.
The average error, rms error and standard deviation are shown in Table II for cities 1, 2 and 3, respectively, with the measured.

### Table II - Average Error, Standard Deviation and RMS Error to the Cities

<table>
<thead>
<tr>
<th>Model</th>
<th>City 1</th>
<th>City 2</th>
<th>City 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Average Error (dB)</td>
<td>Standard Deviation (dB)</td>
<td>RMS Error (dB)</td>
</tr>
<tr>
<td>PE wide angles</td>
<td>1.95</td>
<td>1.54</td>
<td>2.49</td>
</tr>
<tr>
<td>PE narrow angles</td>
<td>3.78</td>
<td>3.72</td>
<td>5.30</td>
</tr>
<tr>
<td>Lee</td>
<td>3.98</td>
<td>3.38</td>
<td>5.22</td>
</tr>
<tr>
<td>Egli</td>
<td>3.35</td>
<td>3.14</td>
<td>4.59</td>
</tr>
<tr>
<td>PE wide angles</td>
<td>4.91</td>
<td>4.41</td>
<td>5.97</td>
</tr>
<tr>
<td>PE narrow angles</td>
<td>4.03</td>
<td>3.02</td>
<td>5.04</td>
</tr>
<tr>
<td>Lee</td>
<td>5.03</td>
<td>4.07</td>
<td>6.47</td>
</tr>
<tr>
<td>Egli</td>
<td>5.56</td>
<td>4.35</td>
<td>7.06</td>
</tr>
<tr>
<td>PE wide angles</td>
<td>2.19</td>
<td>1.83</td>
<td>2.85</td>
</tr>
<tr>
<td>PE narrow angles</td>
<td>3.26</td>
<td>2.28</td>
<td>3.98</td>
</tr>
<tr>
<td>Lee</td>
<td>4.00</td>
<td>2.87</td>
<td>4.92</td>
</tr>
<tr>
<td>Egli</td>
<td>3.68</td>
<td>2.41</td>
<td>4.40</td>
</tr>
</tbody>
</table>

### VI. Conclusion

This paper presented a study of the behavior of the mobile radio signal propagating into urban foliated semi-confined environment. To characterize this region, it was used the measurements obtained in the City 1, 2 and 3, in the State of Pará, where a mobile receiver went into across a road of a region of the Amazon forest. Thus, it was possible to perform a comparative analysis of these measures with the theoretical results obtained by the model proposed here, based on the formalism of parabolic equations for narrow and wide angles of propagation and of two classical models: Lee and Egli.

It was also noted that there was a fast processing the data using parabolic equation model for narrow and wide angles.

In this paper, the refractive index was considered complex.

Through the comparative analysis between these measures with the theoretical results obtained by the model proposed, based on the formalism of parabolic equations for narrow and wide angles of propagation, and the two classical models, Lee and Egli, it was possible to observe that there were not significant differences in the values of the errors using the proposed model for narrow or wide angles of propagation. But, it was possible to observer that a best results were obtained using the parabolic equation model than the models of Lee and Egli.

### References


